How can teachers adapt feedback from mathematical learning environments?

Bastiaan Heeren, **Johan Jeuring**, and Josje Lodder

Open Universiteit Nederland

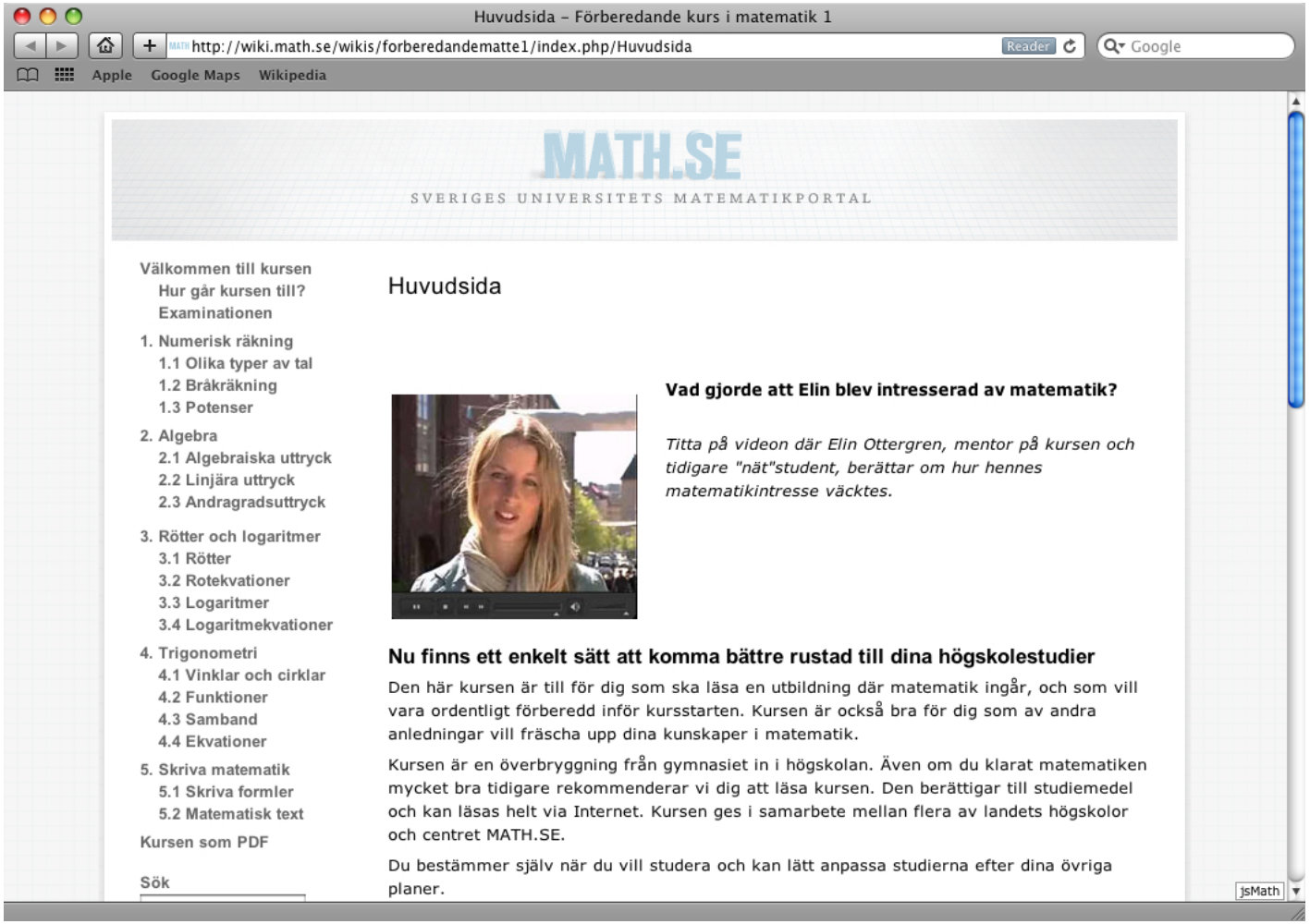
E-learning and mathematics, June 7, 2011

The context

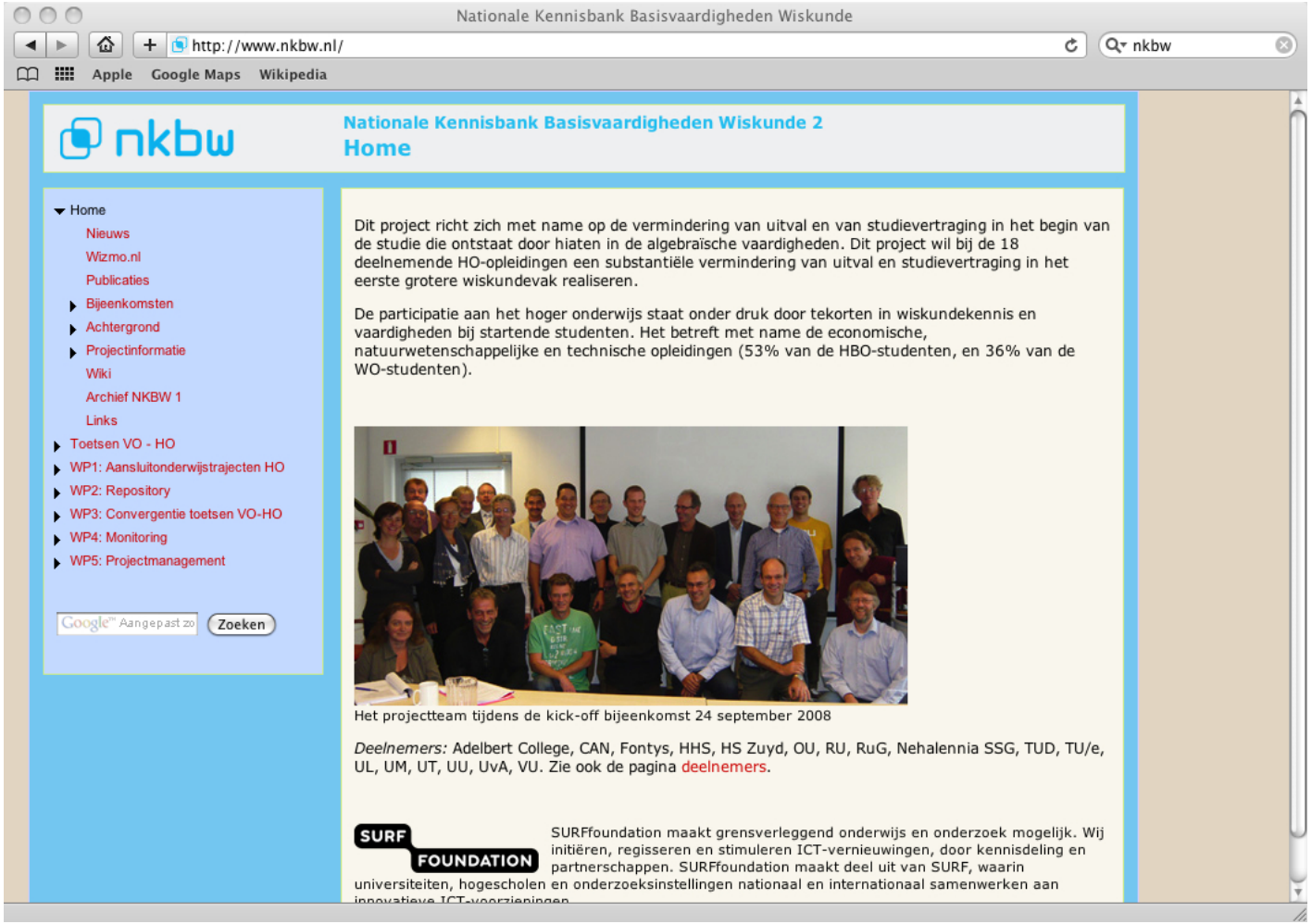
Many universities, all over Europe, offer mathematical bridging courses to help students starting with their university education with mathematics.

Developing bridging courses, in particular on-line courses, is challenging and very time-consuming.

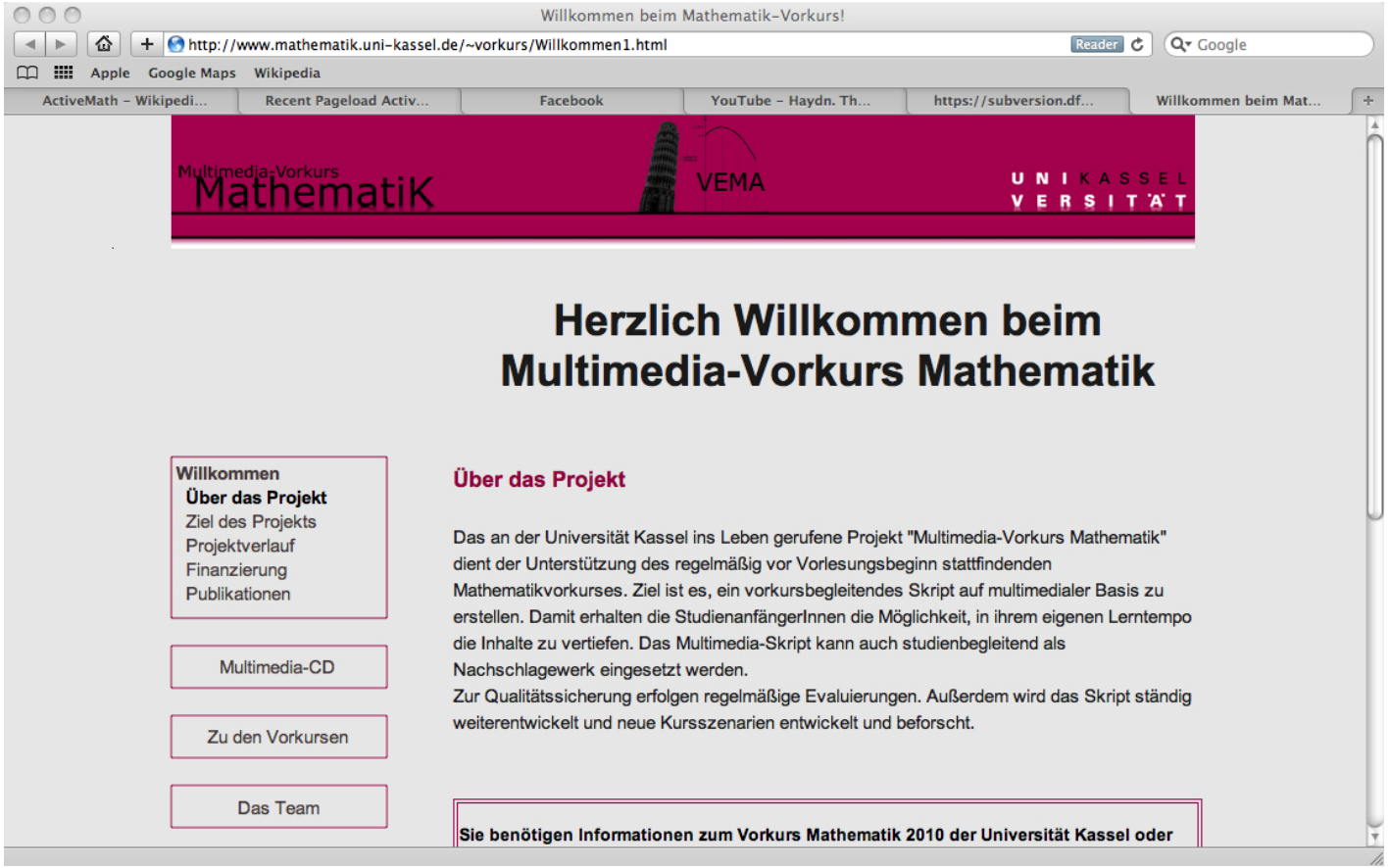
Sweden


Huvudsida – Förberedande kurs i matematik 1
http://wiki.math.se/wikis/forberedandematte1/index.php/Huvudsida
Reader
Google
Apple
Google Maps
Wikipedia
MATH.SE
SVERIGES UNIVERSITETS MATEMATIKPORTAL
Välkommen till kursen
Hur går kursen till?
Examinationen
1. Numerisk räkning
1.1 Olika typer av tal
1.2 Bråkräkning
1.3 Potenser
2. Algebra
2.1 Algebraiska uttryck
2.2 Linjära uttryck
2.3 Andragradsuttryck
3. Rötter och logaritmer
3.1 Rötter
3.2 Rotekvationer
3.3 Logaritmer
3.4 Logaritmekvationer
4. Trigonometri
4.1 Vinklar och cirklar
4.2 Funktioner
4.3 Samband
4.4 Ekvationer
5. Skriva matematik
5.1 Skriva formler
5.2 Matematisk text
Kursen som PDF
Sök
Huvudsida
Vad gjorde att Elin blev intresserad av matematik?
Titta på videon där Elin Ottergren, mentor på kursen och tidigare "nät"student, berättar om hur hennes matematikintresse väcktes.
Nu finns ett enkelt sätt att komma bättre rustad till dina högskolestudier
Den här kursen är till för dig som ska läsa en utbildning där matematik ingår, och som vill vara ordentligt förberedd inför kursstarten. Kursen är också bra för dig som av andra anledningar vill fräscha upp dina kunskaper i matematik.
Kursen är en överbryggning från gymnasiet in i högskolan. Även om du klarat matematiken mycket bra tidigare rekommenderar vi dig att läsa kursen. Den berättigar till studiemedel och kan läsas helt via Internet. Kursen ges i samarbete mellan flera av landets högskolor och centret MATH.SE.
Du bestämmer själv när du vill studera och kan lätt anpassa studierna efter dina övriga planer.
jsMath

The Netherlands



Germany


Willkommen beim Mathematik-Vorkurs!
http://www.mathematik.uni-kassel.de/~vorkurs/Willkommen1.html
Reader
Google
Apple
Google Maps
Wikipedia
ActiveMath – Wikipedi...
Recent Pageload Activ...
Facebook
YouTube – Haydn. Th...
https://subversion.df...
Willkommen beim Mat...
Multimedia-Vorkurs
MathematiK
VEMA
UNIKASSEL VERSITÄT
Herzlich Willkommen beim Multimedia-Vorkurs Mathematik
Willkommen
Über das Projekt
Ziel des Projekts
Projektverlauf
Finanzierung
Publikationen
Multimedia-CD
Zu den Vorkursen
Das Team
Über das Projekt
Das an der Universität Kassel ins Leben gerufene Projekt "Multimedia-Vorkurs Mathematik" dient der Unterstützung des regelmäßig vor Vorlesungsbeginn stattfindenden Mathematikvorkurses. Ziel ist es, ein vorkursbegleitendes Skript auf multimedialer Basis zu erstellen. Damit erhalten die StudienanfängerInnen die Möglichkeit, in ihrem eigenen Lerntempo die Inhalte zu vertiefen. Das Multimedia-Skript kann auch studienbegleitend als Nachschlagewerk eingesetzt werden.
Zur Qualitätssicherung erfolgen regelmäßige Evaluierungen. Außerdem wird das Skript ständig weiterentwickelt und neue Kursszenarien entwickelt und beforscht.
Sie benötigen Informationen zum Vorkurs Mathematik 2010 der Universität Kassel oder

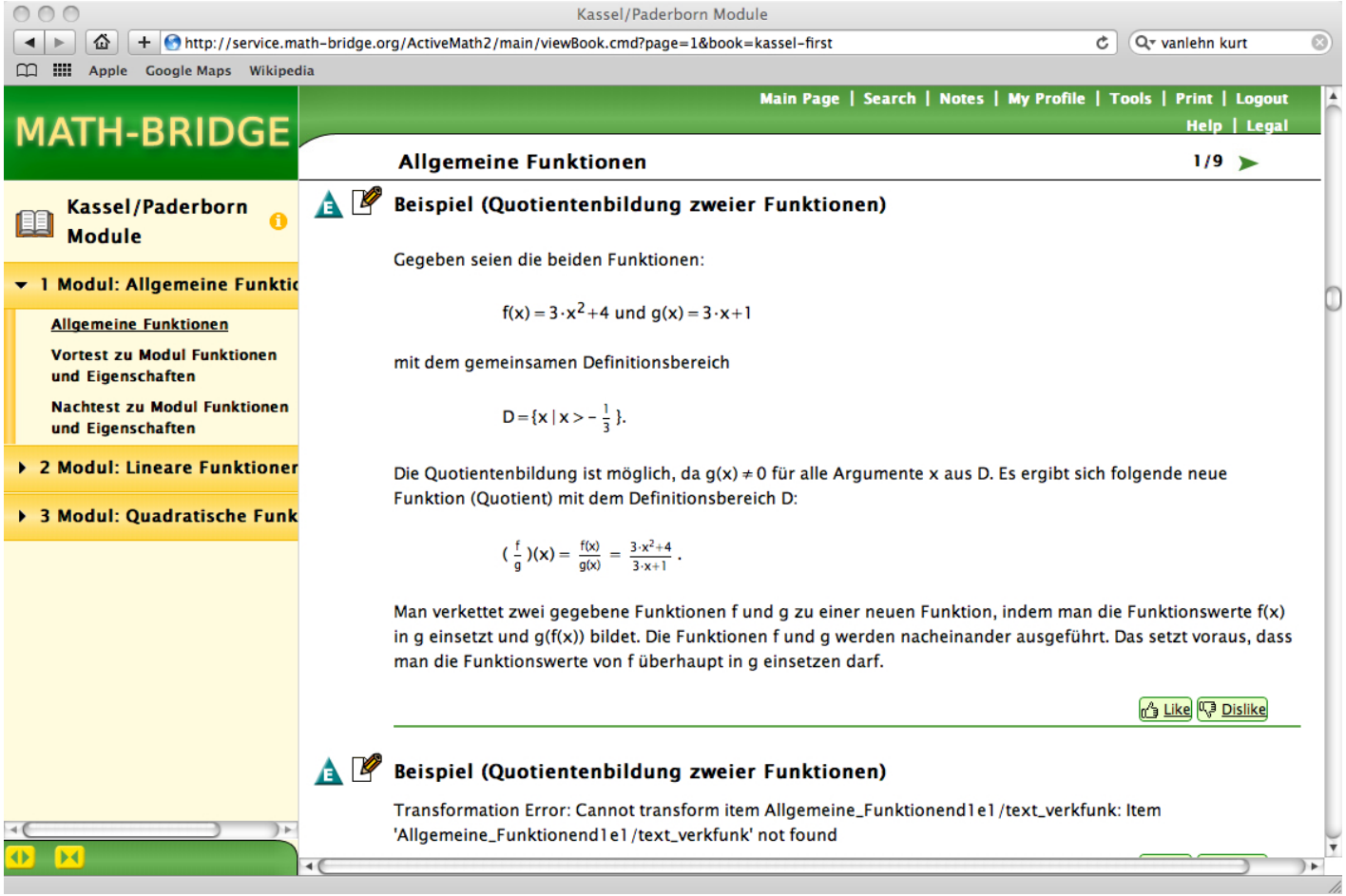
Math-Bridge

- Math-Bridge is a European project, the goal of which is to offer on-line remedial teaching material for mathematics, bridging the gap between high-schools and universities.
- 10+ partners, 6+ countries, 2009-2012.
- Multi-lingual, multi-cultural.
- Open standards, collaborative authoring.
- Using a state-of-the-art intelligent tutoring system for mathematics ActiveMath.

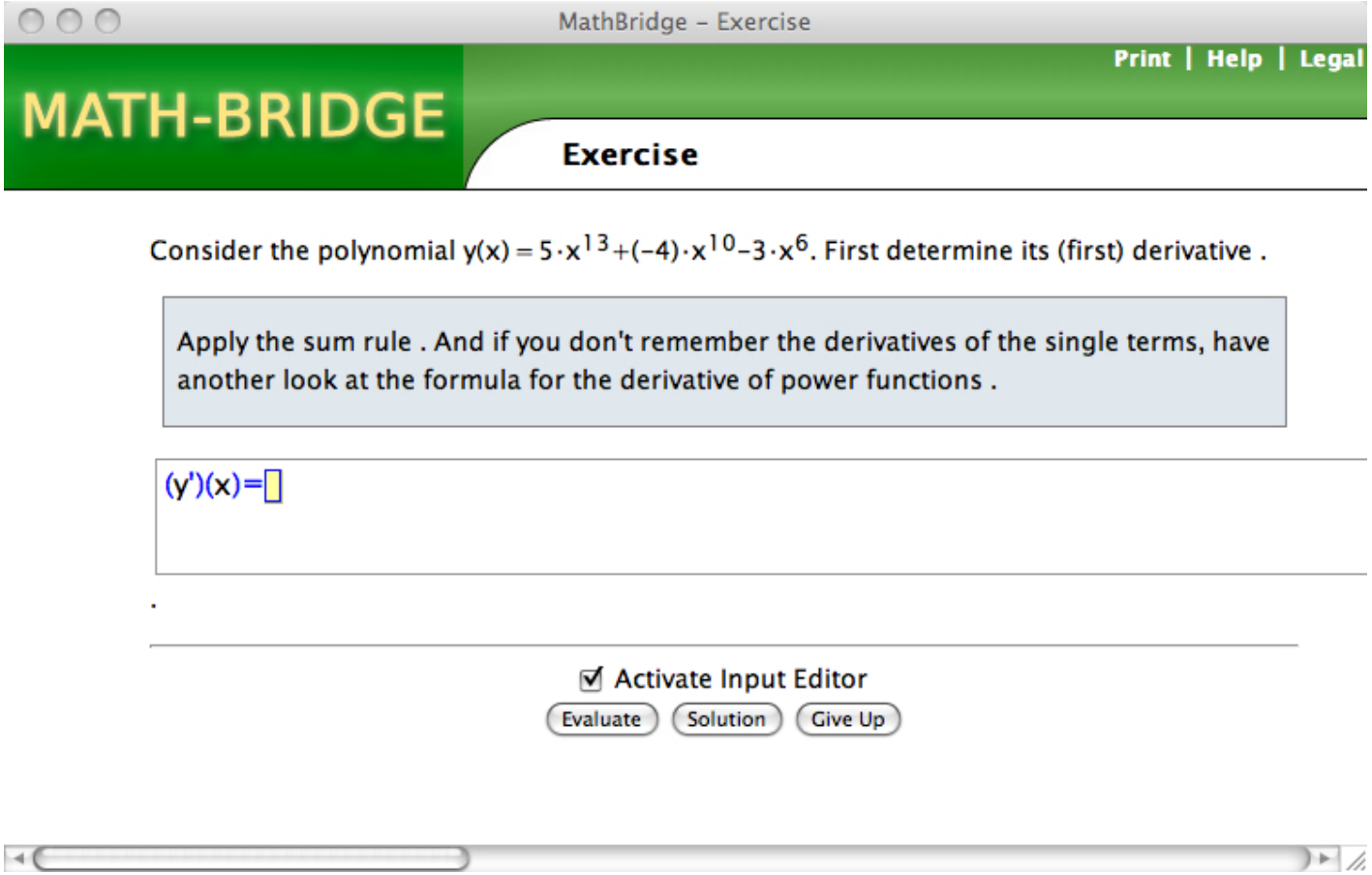
Involved countries

Germany, Austria, Hungary, Spain, Finland, The Netherlands, France.

ActiveMath


Kassel/Paderborn Module
http://service.math-bridge.org/ActiveMath2/main/viewBook.cmd?page=1&book=kassel-first
vanlehn kurt
Apple Google Maps Wikipedia
MATH-BRIDGE
Main Page | Search | Notes | My Profile | Tools | Print | Logout
Help | Legal
Kassel/Paderborn Module
1 Modul: Allgemeine Funktio
Allgemeine Funktionen
Vortest zu Modul Funktionen und Eigenschaften
Nachtest zu Modul Funktionen und Eigenschaften
2 Modul: Lineare Funktionen
3 Modul: Quadratische Funk
Allgemeine Funktionen
1/9
Beispiel (Quotientenbildung zweier Funktionen)
Gegeben seien die beiden Funktionen:
f(x) = 3·x²+4 und g(x) = 3·x+1
mit dem gemeinsamen Definitionsbereich
D = {x | x > − 1/3 }.
Die Quotientenbildung ist möglich, da g(x) ≠ 0 für alle Argumente x aus D. Es ergibt sich folgende neue Funktion (Quotient) mit dem Definitionsbereich D:
(f/g)(x) = f(x)/g(x) = (3·x²+4)/(3·x+1) .
Man verkettet zwei gegebene Funktionen f und g zu einer neuen Funktion, indem man die Funktionswerte f(x) in g einsetzt und g(f(x)) bildet. Die Funktionen f und g werden nacheinander ausgeführt. Das setzt voraus, dass man die Funktionswerte von f überhaupt in g einsetzen darf.
Like
Dislike
Beispiel (Quotientenbildung zweier Funktionen)
Transformation Error: Cannot transform item Allgemeine_Funktionend1e1/text_verkfunk: Item 'Allgemeine_Funktionend1e1/text_verkfunk' not found

ActiveMath

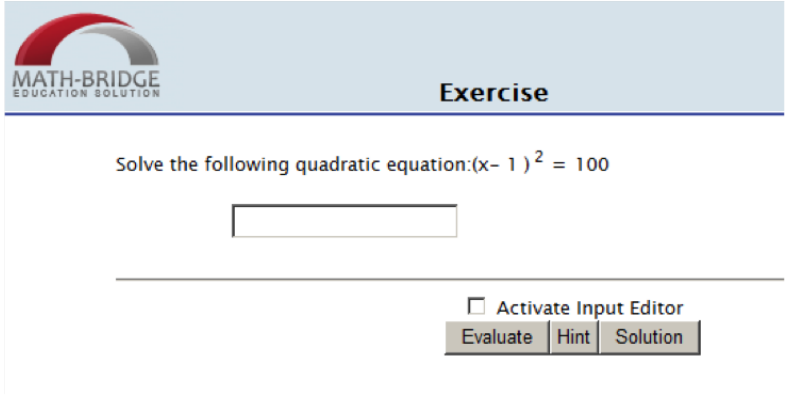

MathBridge – Exercise
Print | Help | Legal
MATH-BRIDGE
Exercise
Consider the polynomial y(x) = 5·x^13+(−4)·x^10−3·x^6. First determine its (first) derivative .
Apply the sum rule . And if you don't remember the derivatives of the single terms, have another look at the formula for the derivative of power functions .
(y')(x)=
Activate Input Editor
Evaluate
Solution
Give Up

ActiveMath – background

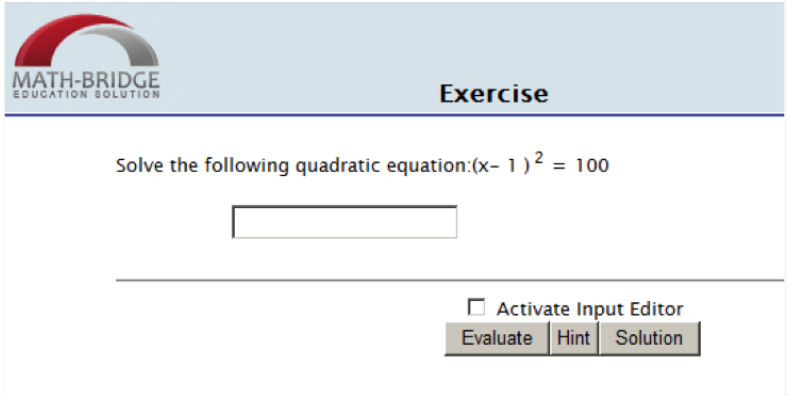
ActiveMath is a state-of-the-art intelligent tutoring system for mathematics, based upon:

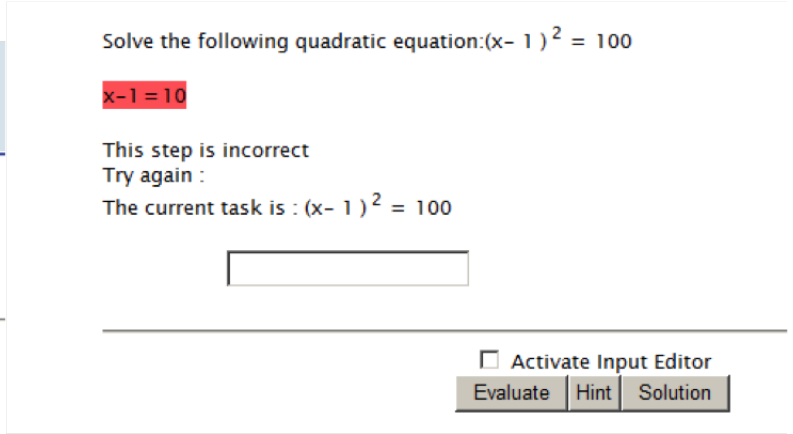
- OMDoc for representing mathematical texts
- Metadata for describing learning objects (exercises, definition, ...)
- Pedagogical metadata
- Student model
- Generic exercise representation, connection to multiple computer-algebra systems/domain reasoners, generic query language

Interactive exercises



Interactive exercises





Interactive exercises

MATH-BRIDGE
EDUCATION SOLUTION

Exercise

Solve the following quadratic equation: $(x-1)^2 = 100$

☐ Activate Input Editor

Evaluate | Hint | Solution

Solve the following quadratic equation: $(x-1)^2 = 100$

$x-1=10$

This step is incorrect
Try again :
The current task is : $(x-1)^2 = 100$

☐ Activate Input Editor

Evaluate | Hint | Solution

The current task is : $(x-1)^2 = 100$

The next correct step is :

$(x-1 = 10) \vee (x-1 = -10)$

☐ Activate Input Editor

Evaluate | Hint | Solution

Interactive exercises

MATH-BRIDGE EDUCATION SOLUTION

Exercise

Solve the following quadratic equation: $(x-1)^2 = 100$

Activate Input Editor

Evaluate | Hint | Solution

Solve the following quadratic equation: $(x-1)^2 = 100$

$x-1=10$

This step is incorrect
Try again :
The current task is : $(x-1)^2 = 100$

Activate Input Editor

Evaluate | Hint | Solution

The current task is : $(x-1)^2 = 100$

The next correct step is :

$(x-1 = 10) \vee (x-1 = -10)$

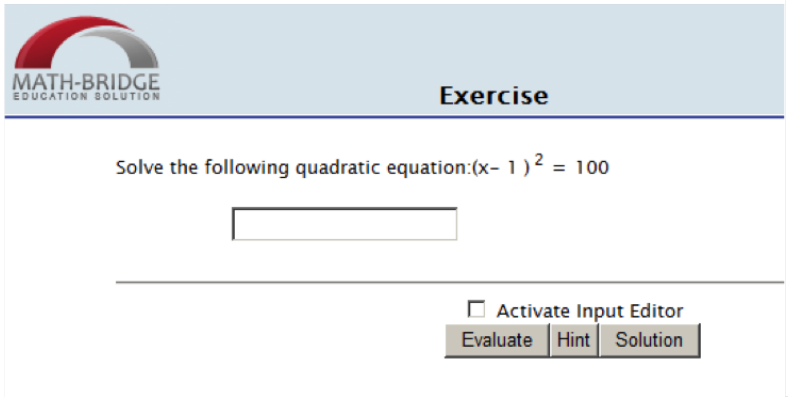
Activate Input Editor

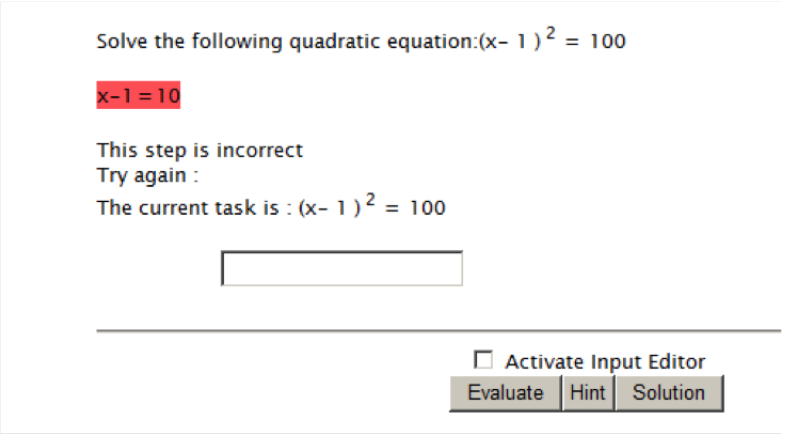
Evaluate | Hint | Solution

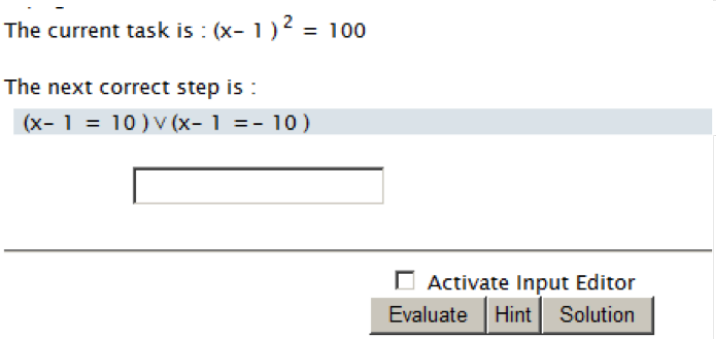
$(x=11) \vee (x=-11)$

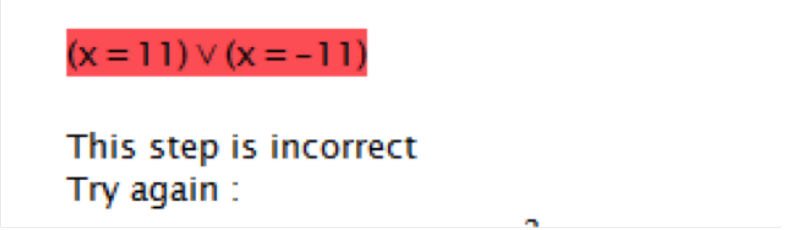
This step is incorrect
Try again :

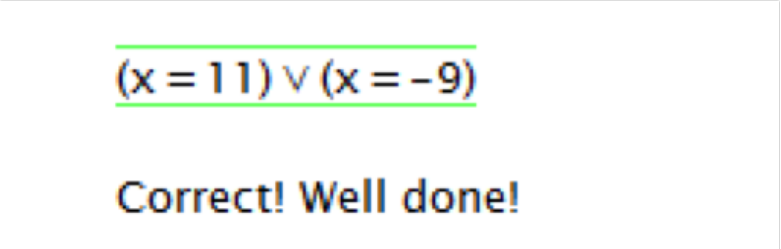
Interactive exercises











The Ideas framework

The Ideas framework provides Feedback services by means of domain reasoners. The feedback services:

- comment on the direction of a step:
  - follows the preferred strategy
  - is an instance of a common error ('buggy rule')
  - cannot be recognized, but is semantically correct
  - is semantically incorrect
- provide a hint on how to proceed (in various level of detail)
- present a worked-out example

Domain reasoners

- Our domain reasoners are based on rewrite strategies for exercises.
- An exercise is solved by subsequently applying rewrite rules following a strategy, until it is solved.
- We have developed strategies for
  - linear, quadratic, and higher-order equations and inequations
  - solving equations and simplifying expressions containing powers, square roots, etc.
  - linear algebra
  - logic
- Our domain reasoners are used in ActiveMath, MathDox, and the DWO.

Hints

At the moment we distinguish the following three level of hints:

- Abstract: Try to solve the quadratic equation.
- Concrete: An equation $y^2 = c$ is solved by calculating the positive and negative square roots
- Bottom-out: $(x-1)^2 = 10$ has the solutions $x - 1 = 10$ and $x - 1 = -10$

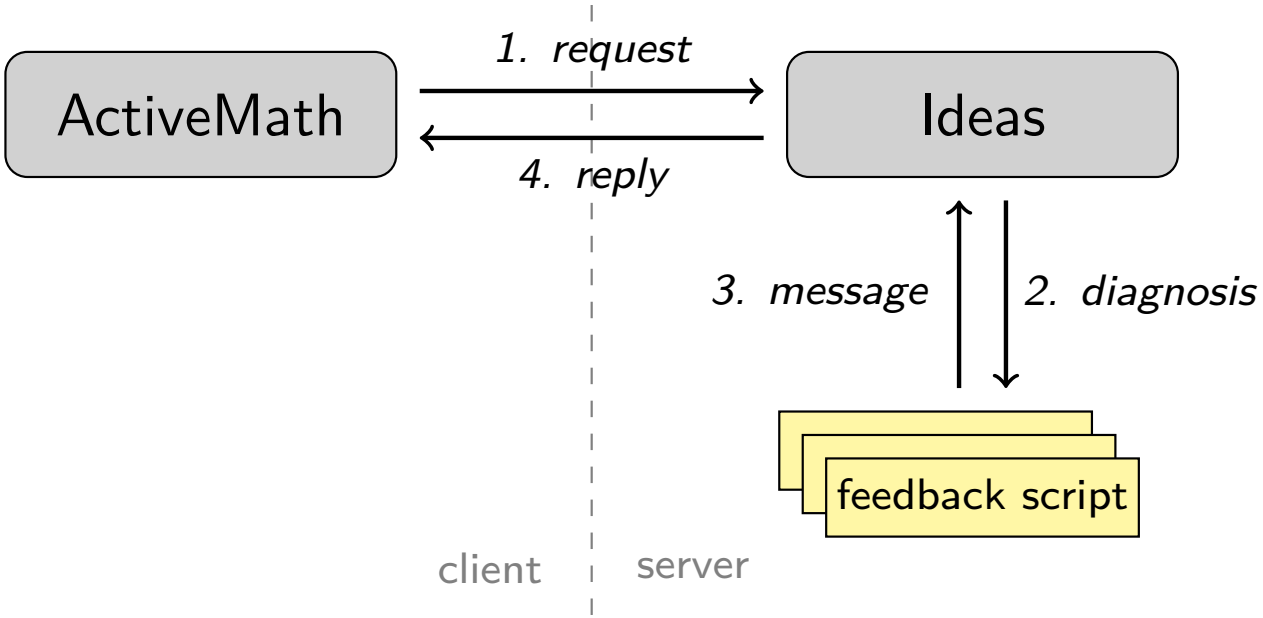
Note that in the last hint, the current expression is used.

Textual feedback

Desirable features for textual feedback:

- support for different levels (abstract, concrete, bottom-out)
- messages available in multiple languages
- can contain dynamic parts such as formulas that depend on the exercise at hand
- should be easy for teachers to adapt feedback

Our approach: feedback scripts



- Server has feedback scripts containing textual messages
- Scripts are used to transform an abstract diagnosis into a message, which is returned to the learning environment
- Possible to select a specific script in a request (e.g. for choosing the language)

Translating rules

$$a \cdot (b + c) \quad \Rightarrow \quad a \cdot b + a \cdot c \qquad \text{(algebra.equations.linear.distr-times)}$$

$$a = b \quad \Rightarrow \quad b = a \qquad \text{(algebra.equations.linear.flip)}$$

```
namespace algebra.equations.linear

text distr-times = {distribute}
text flip = {flip equation around}
```

- All rules are organized in a math taxonomy
- Script provides a translation for all rules of an exercise
- Declaring a namespace prevents long identifier names

Example: worked-out solution

```
text scale-to-one = {divide by @arg1}
```

$$
\begin{aligned}
&4 \cdot (x - 1) = 7 \\
&\quad \Rightarrow \textit{distribute} \\
&4 \cdot x - 4 = 7 \\
&\quad \Rightarrow \textit{bring constants to right} \\
&4 \cdot x = 11 \\
&\quad \Rightarrow \textit{divide by 4} \\
&x = 2\tfrac{3}{4}
\end{aligned}
$$

- Rule translations are used in worked-out solutions
- Attributes (such as `@arg1`) are replaced by dynamic content

Hints at different levels

```
hint abstract = {Use the procedure for solving linear
   equations:  If present, remove parentheses, and
   isolate variable x}
hint concrete = {@expected}
hint bottom-out = {@expected:  this results in @after}
```

- Attribute `@expected` is replaced by the (translation of the) rule suggested by the strategy
- Attribute `@after` represents the term after application of the expected rule
- Feedback texts can be further tailored for a specific rule-level combination
- OpenMath is used for encoding mathematical objects

Feedback at different levels

```
feedback noteq = {This is incorrect.}
feedback buggy = {This is incorrect.  @recognized}

feedback ok    = {Well done!  You used @recognized}
feedback same  = {This is correct.}

# Messages for buggy rules
text buggy.distr-times-plus = {Did you try to use
   distribution?  One term was not multiplied.}
text buggy.negate-one-side = {It seems you have negated
   the terms on one side only.}
```

- The script contains messages for each type of diagnosis: *buggy*, *noteq*, *ok*, *same*, *detour*, and *unknown*
- Messages can again be specialized for the levels

More features

- String definitions and an include mechanism provide a way to reuse text fragments
- Conditionals make it possible to report tailor-made feedback messages for specific cases
- Many more attributes help to enrich the messages with dynamic content, including attributes for the number of steps remaining or the subexpression that is replaced
- Also strategy labels can be used to construct messages
- Feedback scripts can be analyzed for correctness:
  - Syntax errors are reported
  - Unknown attributes and non-existing rule identifiers result in warnings
  - Scripts can be tested for completeness, i.e., whether all cases are covered by the script

Conclusions

- To bridge the gap in mathematical competencies between schools and higher education, the European Math-Bridge project provides on-line mathematics learning facilities.
- Interactive exercises are important for mathematical learning environments
- Interactive exercises need feedback facilities
- Feedback texts need:
  - support for different levels (abstract, concrete, bottom-out)
  - messages in multiple languages
  - dynamic parts such as formulas that depend on the particular exercise
- Teachers can adapt or translate feedback by changing the feedback scripts, which are separate from the domain reasoner code. Some more tool support is needed