

Skills & Technology

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2011-06-06

ELAM

[Faculteit Bètawetenschappen]

FISME Freudenthal Institute for Science and Mathematics Education]



Universiteit Utrecht

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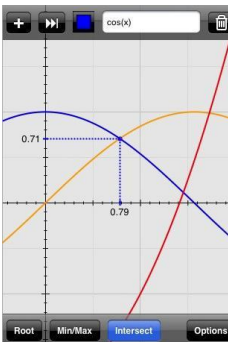
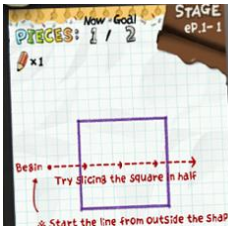
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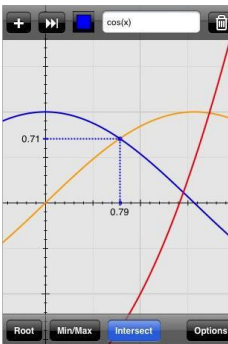
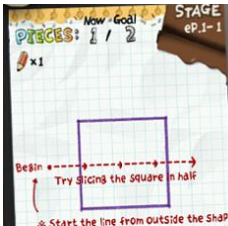
Outline

1. Introduction on tools and tool use
2. Paper-and-pencil skills and tool skills / techniques
3. Basic skills and higher order skills / thinking skills
4. Pedagogical skills and technical skills
5. Conclusion



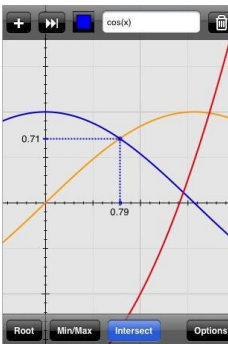
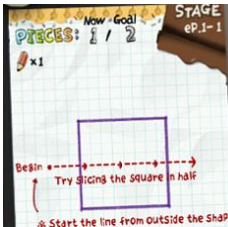
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1. Introduction on tools and tool use

- ICT can improve skill acquisition and performance, including both basic skills and higher order skills.
- The relationship between tool use and skill acquisition, however, is complex and subtle, and achievement depends on didactical setting (teacher skills!), tasks, and opportunities for transfer

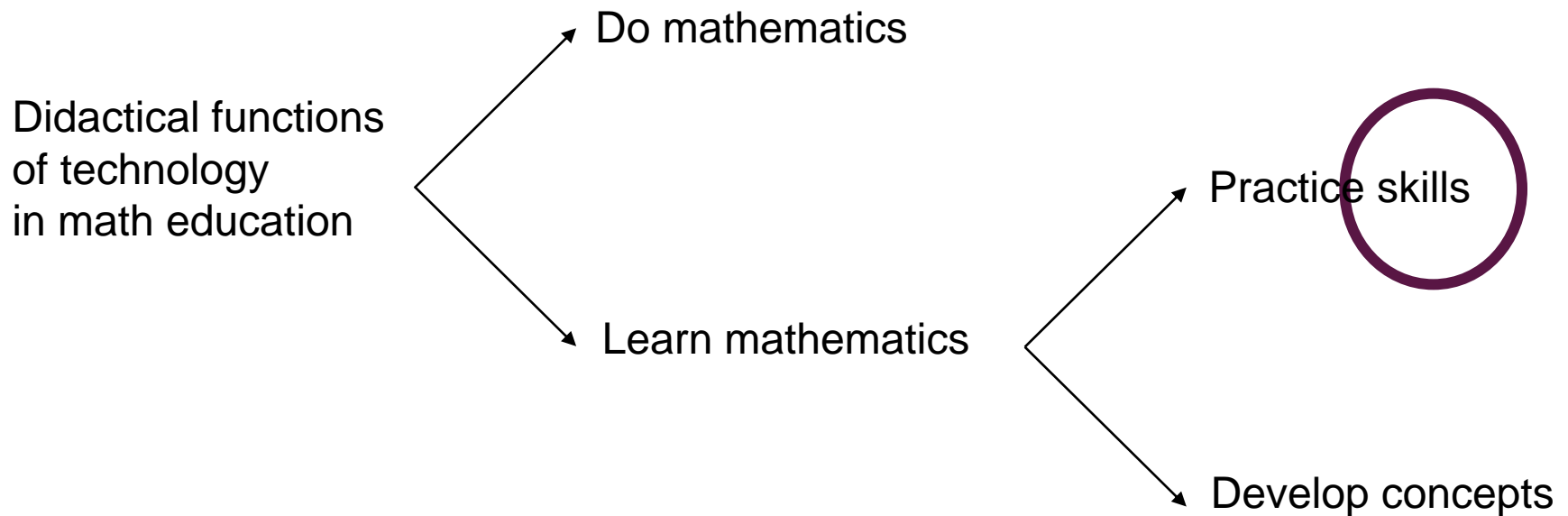
1. Introduction on tools and tool use

Tools matter!



1. Introduction on tools and tool use

Didactical functions of digital tools



Drijvers, P. & Zwaneveld, B. (2008). Van knoppen naar kennis. *Handboek Vakdidactiek Wiskunde*. www.elwier.nl

1. Introduction on tools and tool use

Quotes like these are widespread:

“Presently the benefits of software in education are apparent and widely acknowledged.”

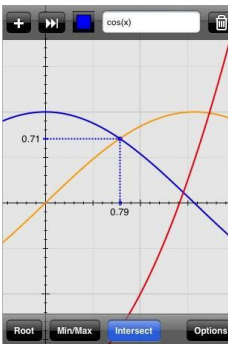
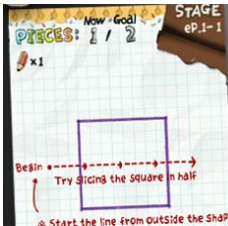
But how about this one:

“Presently the benefits of *paper and pencil* in education are apparent and widely acknowledged.”

We might need to be more specific when we consider technology in education....

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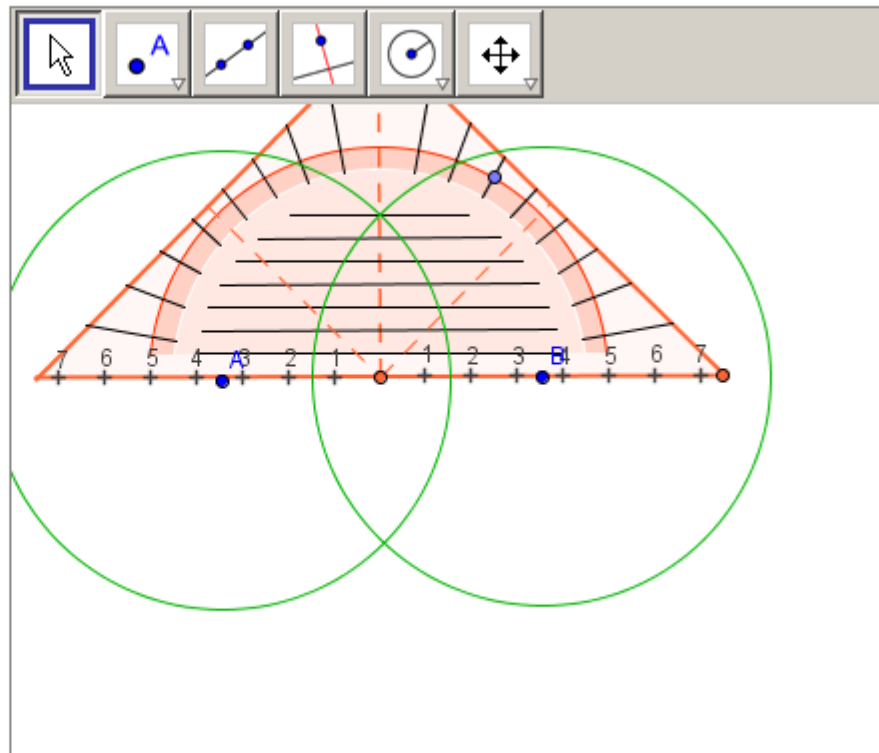
2. Paper-and-pencil skills and tool skills / techniques

- Example Geogebra: www.fi.uu.nl/dwo/dpict

Gelijke afstand tot twee punten

Gegeven zijn de punten A en B .

- Teken alle punten op afstand 5 van A en alle punten op afstand 5 van B .
- Hoeveel punten liggen er op afstand 5 van zowel A als B ?
Antwoord: punten
- Teken alle punten op afstand 6 van zowel A als B en alle punten op afstand 4 van zowel A als B .
- De punten die je net hebt getekend liggen op een lijn. Wat is de hoek van deze lijn met de lijn AB ?
Antwoord: graden



Sietske Tacoma, Peter Boon

kijk na ✓

2. Paper-and-pencil skills and tool techniques

- From book to screen:

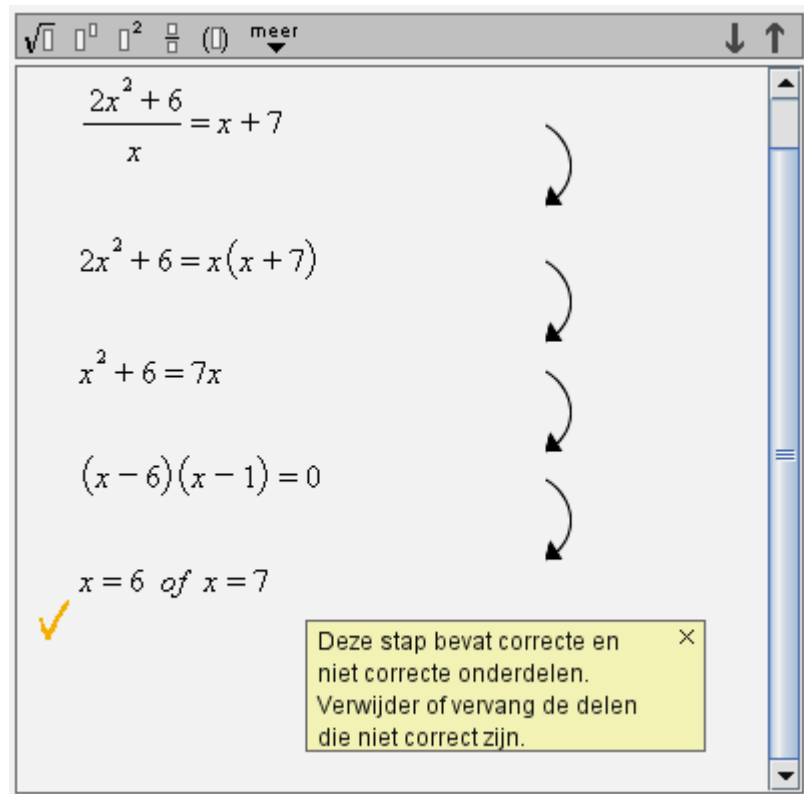


Je kunt de opgave ook oefenen met **vergelijkingen 2**.

7 Los exact op.

a $\frac{x^2 + 3}{2x} = x - 1$

b $\frac{4^x - 2^x - 6}{2^x - 4} = 0$



$\frac{2x^2 + 6}{x} = x + 7$

$2x^2 + 6 = x(x + 7)$

$x^2 + 6 = 7x$

$(x - 6)(x - 1) = 0$

$x = 6$ of $x = 7$

Deze stap bevat correcte en niet correcte onderdelen. Verwijder of vervang de delen die niet correcte zijn.

<http://www.fi.uu.nl/dwo/gr-pilot/>

2. Paper-and-pencil skills and tool techniques

- From screen to notebook: transfer between computer tool techniques and paper-and-pencil techniques

A screenshot of a digital math tool interface. The toolbar at the top includes icons for square root, power, fraction, parentheses, plus, minus, multiply, divide, percent, and a red 'Terug' button. The main display area shows the following steps to solve the equation $6x - 6 = 2x + 16$:

$$6x - 6 = 2x + 16$$
$$6x = 2x + 22 \quad \left. \begin{array}{l} \\ \end{array} \right\} + 6$$
$$4x = 22 \quad \left. \begin{array}{l} \\ \end{array} \right\} - 2x$$
$$x = 5\frac{1}{2} \quad \left. \begin{array}{l} \\ \end{array} \right\} \div 4$$

Handwritten mathematical work showing the steps to solve the equation $2x + 8 = -4x - 16$:

$$3. \quad 2x + 8 = -4x - 16$$
$$-8 \quad \downarrow \quad 2x = -4x - 24 \quad -8$$
$$+4x \quad \downarrow \quad 6x = -24 \quad +4x$$
$$\div 6 \quad \downarrow \quad x = -4 \quad \div 6$$

2. Paper-and-pencil skills and tool techniques

- Such transfer is not always straightforward:

F1+ Tools	F2+ Algebra	F3+ Calc	F4+ Other	F5 Pr9m10	F6+ Clean Up
--------------	----------------	-------------	--------------	--------------	-----------------

```
■ solve(x^2 + b·x + 1 = 0, x)
x =  $\frac{\sqrt{b^2 - 4} - b}{2}$  or x =  $-\frac{\sqrt{b^2 - 4}}{2}$ 
solve(x^2+b*x+1=0,x)
MAIN          RAD AUTO      FUNC      1/30
```

$$\begin{aligned} \text{nulpunten} &= \frac{\sqrt{b^2 - 4} - b}{2} \quad \text{en} \quad \frac{-\sqrt{b^2 - 4} + b}{2} \\ \text{midden} &= -\frac{1}{2} b. \end{aligned}$$

Drijvers, P. (2003). *Learning algebra in a computer algebra environment*.
www.fi.uu.nl/~pauld/dissertation

2. Paper-and-pencil skills and tool techniques

Solving an equation is not straightforward either:

An equation is solved with respect to an unknown

An equation needs an = sign

A solution can be an expression

Notice the scope of the square root sign

'solve with respect to x ' = 'express x in terms of b '

The screenshot shows a TI-84 Plus calculator interface. At the top, there are function keys: F1-Tools, F2-Algebra, F3-Calc, F4-Other, F5-Pr3mid, and F6-Clean Up. The main display area shows the command `■ solve(x^2 + b·x + 1 = 0, x)`. Below this, the solutions are displayed as $x = \frac{\sqrt{b^2 - 4} - b}{2}$ or $x = \frac{-\sqrt{b^2 - 4} - b}{2}$. At the bottom of the screen, the status bar shows `MAIN`, `RAD AUTO`, `FUNC`, and `1/30`. Several purple arrows point from text annotations to specific elements: one points to the equals sign in the equation, another to the square root symbol, a third to the variable x in the denominator of the solution, and a fourth to the entire solution expression.

Intermezzo Instrumentation: Artefact and instrument

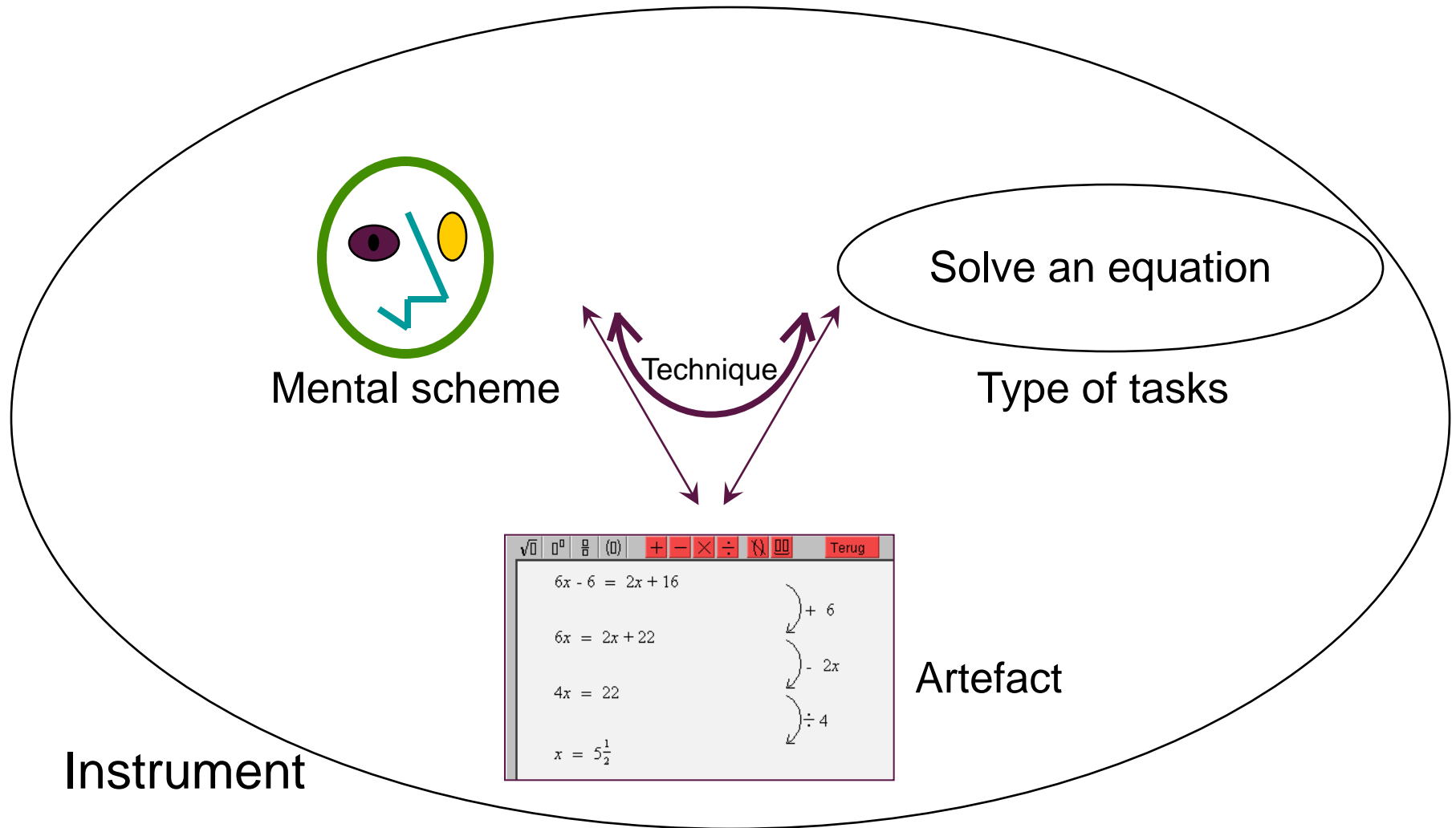
- A good artefact does not imply nice music!
- Instrument = Artefact + Schemes
- Scheme = Technique + Concepts



Schemes and techniques

- A *technique*: a concrete way to carry out a task using an artefact
-> what you **do** with the artefact
- A *scheme*: a method for solving a class of problems, including the corresponding insights
-> what you **think** while carrying out a technique
- What you think determines what you do, but the artefact's affordances and constraints guide your thinking

Instrumentation: a sketch



Instrumentation: conclusion

- There is a close relationship between
 - Techniques and representations that tools invite and offers, and
 - Mental schemes that students develop
- This relation has to be taken into account and exploited for educational goal
- Tools + Task + Technique ->
Cognitive development

Drijvers, P. (2007). Instrument, orkest en dirigent: een theoretisch kader voor ICT-gebruik in het wiskundeonderwijs. *Pedagogische Studiën*, 84 (5), 358-374.

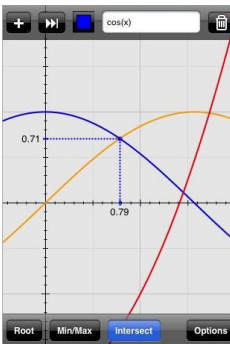
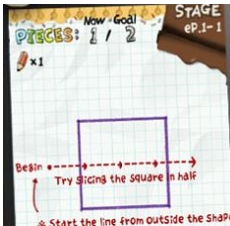
2. Paper-and-pencil skills and tool techniques

Conclusion:

- Paper-and-pencil skills can benefit from the work with technology.
- There is a subtle relationship between student skills, tool use, and paper-and-pencil use.
- Transfer needs to be considered carefully.

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3. Basic skills and higher order skills

cTWO 'thinking activities' in mathematics education:

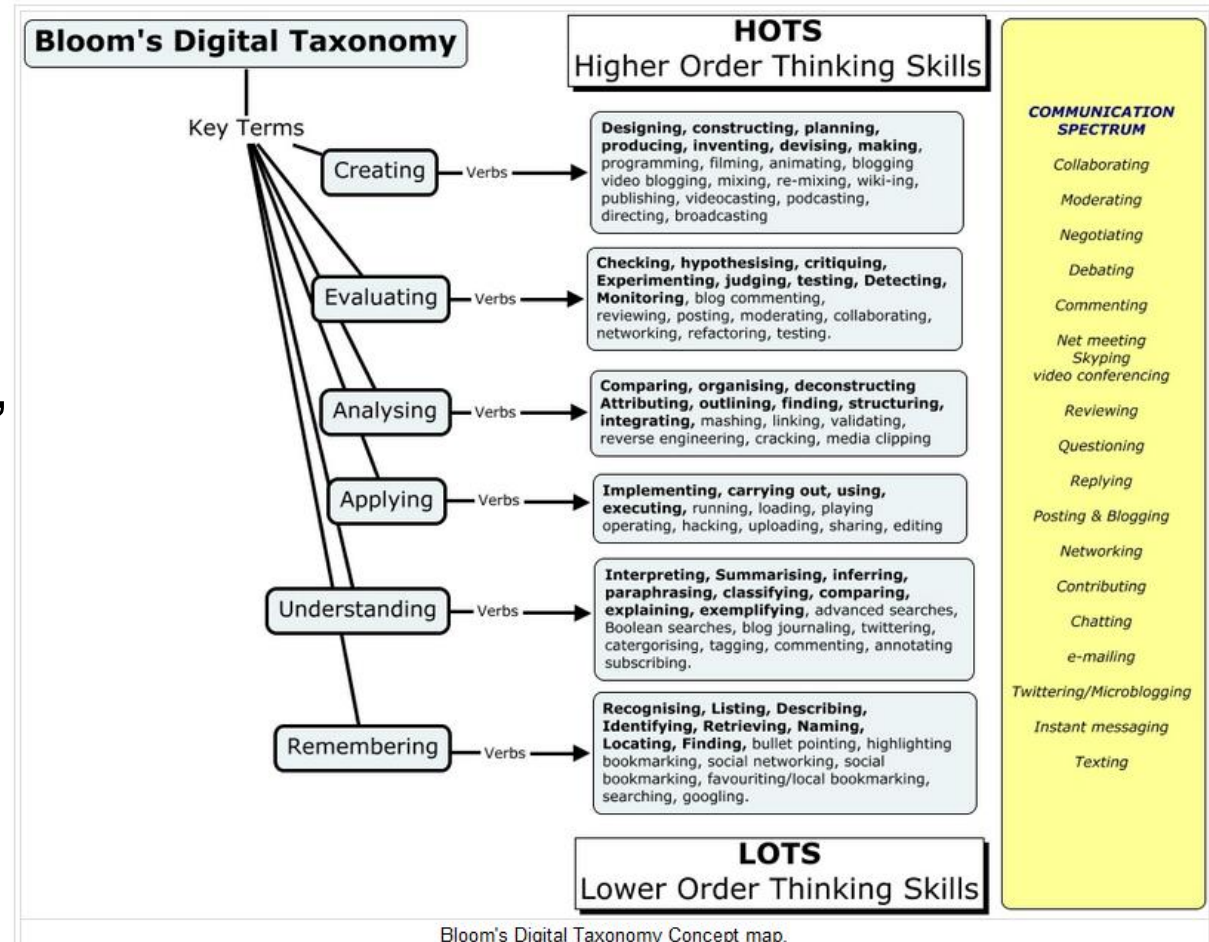
- Modelling and algebrizing
- Ordening and structuring
- Analytical thinking and problem solving
- Algebraic manipulation
- Abstraction
- Logical reasoning and proving

? Can technology help here, or is it just for the rote learning of basic skills?

3. Basic skills and higher order skills

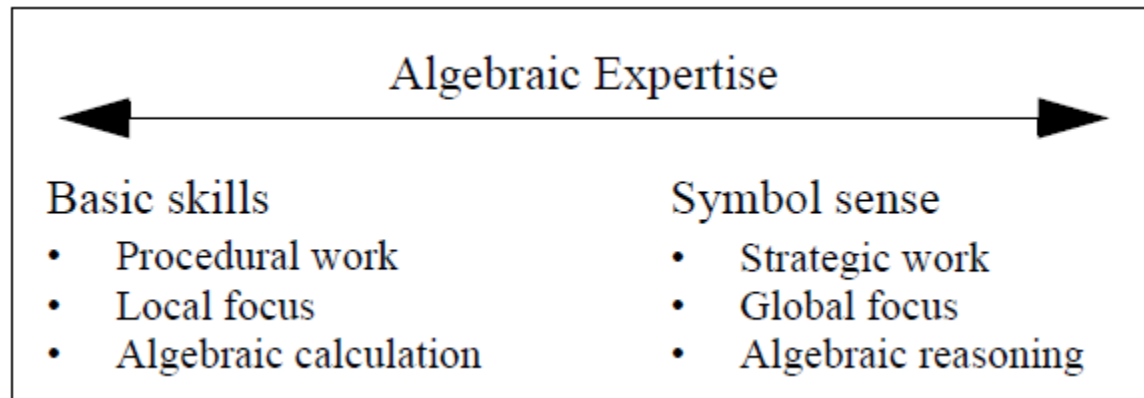
Bloom's Digital Taxonomy Summary Map

Technology in education can invite both 'HOTS' and 'LOTS'!



3. Basic skills and higher order skills

- LOTS & HOTS specifically for algebra:



Drijvers, Kindt, & Goddijn, 2010

Secondary Algebra Education

Secondary Algebra Education

Revisiting Topics and Themes and Exploring the Unknown

Paul Drijvers (Ed.)



Paul Drijvers (Ed.)

SensePublishers

3. Basic skills and higher order skills

B=A*C" van Charlotte Franssen

$(5x^2 - 90)^2 = 225x^2$

$(5x^2 - 90)(5x^2 - 90) = 225x^2$

$25x^4 - 900x^2 + 8100 = 225x^2$

$25x^4 - 1125x^2 + 8100 = 0$

$x^4 - 45x^2 + 324 = 0$

$x^4 - 45x^2 = -324$

C" van Charlotte Franssen

$x^4 - 45x^2 = -324$

$5x^2 - 90 = 15x$ of $5x^2 - 90 = -15x$

$5x^2 - 15x - 90 = 0$ of $5x^2 + 15x - 90 = 0$

$x^2 - 3x - 18 = 0$ of $x^2 + 3x - 18 = 0$

$(x - 6)(x + 3) = 0$ of $(x + 6)(x - 3) = 0$

$x = 6$ of $x = -3$ of $x = -6$ of $x = 3$

De vergelijking is correct opgelost.

A red arrow points from the expansion step in the left window to the factoring step in the right window.

3. Basic skills and higher order skills

- A lot of LOTS, a lack of HOTS:

$v \cdot \sqrt{u} = 1 + 2v \cdot \sqrt{1+u}$
 $v = (1 + 2v\sqrt{1+u}) \cdot \frac{1}{\sqrt{u}}$
 $v = \frac{1 + 2v\sqrt{1+u}}{\sqrt{u}}$
 $v = \frac{\sqrt{1+u} \left(\frac{1}{\sqrt{1+u}} + 2v \right)}{\sqrt{u}}$

Δt	Technique	Interpretation
0		The student isolates v by dividing by \sqrt{u} ($\neg V$)
3	Rewrite	
4	Rewrite	
5	Factor	The student realizes $2v$ poses a problem and tries to lose this term by factoring
9		The student tries a next step, backtracks, tries again and finally gives up

Bokhove & Drijvers, 2010

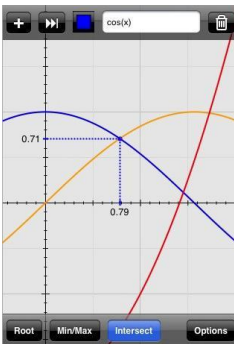
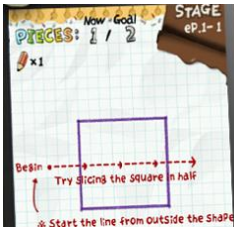
3. Basic skills and higher order skills

Conclusion

- Can technology invite higher order skills, or is it just for the rote learning of basic skills?
- Yes, it can, but this largely depends on the type of tools, the type of tasks, and the educational setting

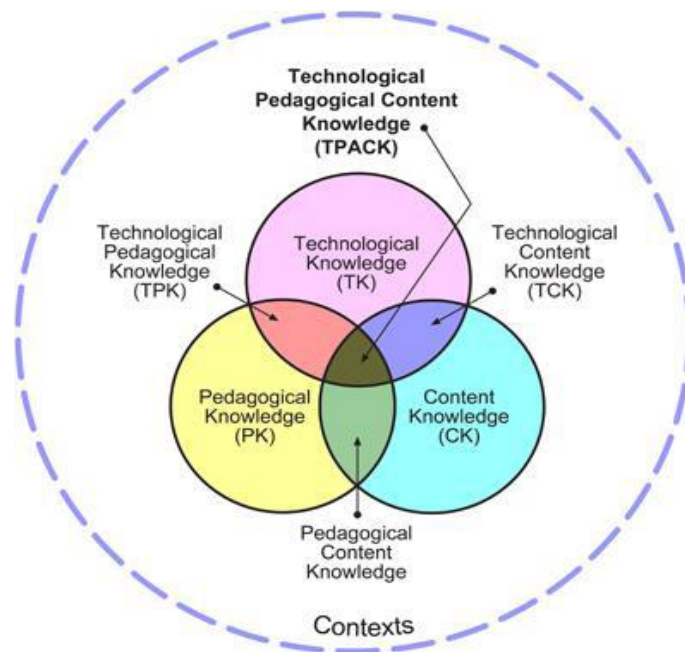
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4. Pedagogical and technical skills

- In the development of LOTS & HOTS with technology, and in the transfer to paper-and-pencil, teachers play a crucial role
- This requires both technological and technical skills:



www.tpack.org

4. Pedagogical and technical skills

- An example: transfer screen - board

The screenshot shows a web browser window titled "Digitale Wiskunde Oefenomgeving - Mozilla Firefox" with the URL <http://www.fi.uu.nl/dwo/prootool/en/frameset.html>. The page is for "Digital activities PROO Tool Use" and is titled "2. Practising with Algebra Arrows". It features a "WisWeb" logo for the Freudenthal Institute and a user login for "Paul Drijvers".

Task 9

a. Make an arrow chain which provides the following input - output pairs.

input	0	1	2	3	4
output	3	5	7	9	11

b. Explain your approach.

c. Make a second, different arrow chain, which does provide the same value table.

The interface includes an "In-/Output" field, a vertical menu of operations (+3, -3, x3, /3, 1/..., √..., ...2), and a navigation bar with buttons for Back, Forward, Table, Graph, and Clear. A progress indicator at the bottom shows 9 steps, with the 9th step highlighted.

4. Pedagogical and technical skills

- Student work:

Resultaten
Resultaten van de Activiteit "Oefenen met AlgebraPijlen" van Joris en Joost

Opdracht 9

a.
Maak een pijlenketting waarvoor geldt:

invoer 0 1 2 3 4
uitvoer 3 5 7 9 11

b.
Leg uit hoe je dit hebt aangepakt.

uitproberen

c.
Maak een tweede, andere pijlenketting die wel dezelfde uitvoertabel geeft als de eerste ketting.

In-/Uitvoer

Bewerkingen

- + 3
- 3
- x 3
- / 3
- 1/...
- √...
- ... 2

Terug Heen

Tabel
 Grafiek

Wis

Diagram illustrating a flowchart with input and output tables and operations:

Input 1: 0, 1, 2, 3, 4, 5, 6, 7
Output 1: 3, 5, 7, 9, 11, 13, 15, 17

Input 2: 0, 1, 2, 3, 4, 5, 6, 7
Output 2: 3, 5, 7, 9, 11, 13, 15, 17

Operations: x 2, + 3, + 1.5, x 2

Progress indicator: 1 2 3 4 5 6 7 8 9

Sluiten

Java Applet Window

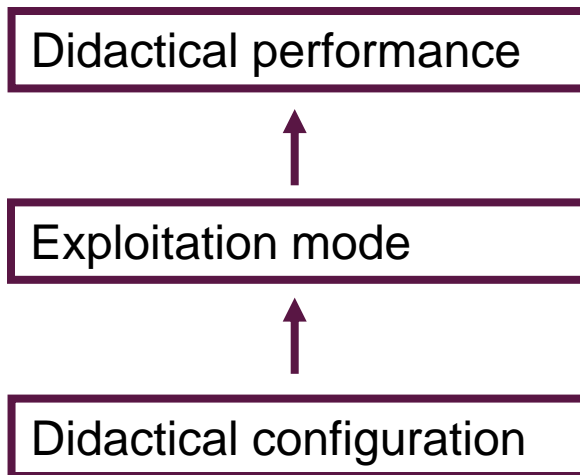
4. Pedagogical and technical skills

- Teacher work:



Intermezzo: orchestration

Instrumental orchestration



Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75(2), 213-234.

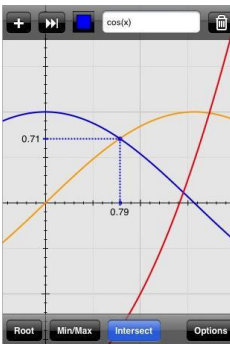
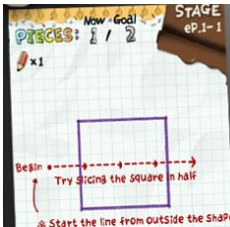
4. Pedagogical and technical skills

Conclusion:

- Teachers remain crucial in exploiting the opportunities offered by technology
- Teachers need both pedagogical and technical skills to do so
- This requires a process of professional development

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